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Parameter estimation of dynamical systems via a chaotic ant swarm

Haipeng Peng, Lixiang Li,* Yixian Yang, and Fei Liu

Information Security Center, State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, P.O. Box 145, Beijing 100876, China; Key Laboratory of Network and Information Attack and Defence Technology of MOE, Beijing University of Posts and Telecommunications, Beijing 100876, China; and National Engineering Laboratory for Disaster Backup and Recovery, Beijing University of Posts and Telecommunications, Beijing 100876, China

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Through the construction of suitable objective function, the parameter estimation of the dynamical system could be converted to the problem of parameter optimization. Based on the chaotic ant swarm optimization approach, we investigate the problem of parameter optimization for the dynamical systems in the presence of noise. We systematically analyze the basic relationships among the complexity of objective function, the length of time series, and the performance of the searching algorithm. Furthermore, we consider the effect of measurable additive noise on the objective function. Numerical simulations are also provided to show the effectiveness and feasibility of the proposed methods.

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I. INTRODUCTION

The problem of modeling from time series is known as "system identification" and "reconstruction of dynamical systems" [1]. Two types of models, that is to say, gray box model and black box model, are common in the field of system identification. For the gray box model, although the peculiarities of what is going on inside the system are not entirely known, a certain model based on both insight into the system and experimental data is constructed. However, there are still a number of unknown free parameters which can be estimated using system identification. For the black box model, no prior model is available. Parameter estimation (identification) problems deal with the reconstruction of unknown functions or geometric objects appearing as parameters in systems of differential equations. In general, if the basic mathematical model of the dynamical system is known, we only need to estimate the unknown parameters of the dynamical system equations. Consequently, the gray box problem of system identification usually equates to the problem of parameter estimation of the specified system.

From the aspects of estimation methods, the least-squares method and its variants are the basic approaches for the parameter estimation. It was first developed by Gauss, and it could be used in the linear and nonlinear systems [2]. The Kalman filter method is one of the widely used estimation methods [3]. Synchronization based methods are also proposed to identify the parameter [4–6]. Recently, many intelligent optimization algorithms are developed for the parameter identification such as tabu search, genetic algorithms [7], neural network [8], and particle swarm [9]. The parameter estimation process utilizing heuristic optimization algorithms is based on the objective function, and here optimization, in mathematics, refers to choosing the best element from some set of available alternatives. Through the construction of suitable objective function, the problem of parameter estimation of the dynamical system could be converted to that of parameter optimization. However, there are still few discussions about the influence of objective function on the parameter estimation, especially in presence of noise.

In recent years, there is a significant interest in developing rapidly converging optimization algorithms based on animal foraging routines. Based on the biological two-bridge experiment of ants, Dorigo and Eberhart developed random models called ant colony optimization (ACO) algorithms [10]. One may now find applications of these algorithms in different areas such as robotics, objects clustering [11], communication networks [12], and combinatorial optimization [13]. Inspired by a biological experiment of ant's chaotic behavior, we recently gave a mathematical model called chaotic ant swarm (CAS) optimization algorithm in Ref. [14]. It is a derivative-free method. In CAS, the individuals of the ant colony exchange information and benefit from their own experience together with the experiences of other individuals, while exploring promising areas of the search space. The model and mechanism of CAS are different from those of ACO. The CAS algorithm has been applied in different areas such as fuzzy system identification [15], economic dispatch [16], computation of multiple global optima [17], data clustering, and parameter identification [18].

In our previous work [18], we studied the parameter identification problem of chaotic system. But we did not study the parameter identification problem of other dynamical systems. Usually the objective function and noise have important impact on the estimation performance. However in Ref. [18], we did not study the effects of objective function and noise. So in this paper, we first discuss the influence of objective function on the estimation performance in the process of parameter estimation. The relationships among the complexity of objective function, the length of time series, and the performance of the searching algorithm are systematically analyzed when the dynamical system is in one of the four states—stability, incipiently unstable, periodicity, and chaos. Second, we study the effects of noise on the objective

*li_lixiang2006@yahoo.com.cn

function. The existence of noise increases the numbers of local minima for the objective function and adds the difficulty of parameters estimation. Finally, based on the chaotic ant swarm algorithm, numerical simulations about parameter estimation of dynamical systems are given to illustrate the effectiveness and feasibility of the proposed methods.

II. CHAOTIC ANT SWARM ALGORITHM

In recent years, ant colonies, and more generally social insect societies, have always fascinated human beings. What particularly strikes the occasional observer as well as the scientist is the high degree of societal organization that these insects can achieve in spite of very limited individual capabilities. As a result of this organization, ant colonies can accomplish complex tasks that in some cases far exceed the individual capacities of a single ant. Social ants have self-organization and information transfer (communication) behaviors. Ant societies are a particular (though very extended) class of social organization. The global patterns of behavior are the result of emergent phenomena not reducible to the properties of individuals. In the past few years, such emergent behavior was reported in relation with chaotic dynamics in *Leptothorax* ant colonies. Global oscillations of colony activity were reported together with the observation that individual behavior can be characterized by means of low-dimensional strange attractors [19]. The study of ant chaotic behavior and of their self-organizing capacities brings great interest from computer scientists to develop models of distributed organization which are useful to solve difficult optimization and distributed control problems. In the following, we give the detailed chaotic ant swarm algorithm based on some biological observations and investigations on the chaotic and self-organizing behaviors of ants.

A. Chaotic and self-organization behaviors of ants

Chaos in insect behavior was first reported by Cole from his experimental studies on activity cycles in ants of the species *Leptothorax allardycei* [19]. Cole used a solid-state automatic digitizing camera to perform a careful experimental study of individual isolated ants and ant colony dynamical behaviors. The investigations by Cole showed that "the ant behavior may be chaotic. The attractor of the movement activity of single, isolated *Leptothorax longispinosus* ants has a small, noninteger dimension characteristic of low-dimensional chaos. The activity of entire colonies of ants yields an integer dimension that is consistent with periodicity in activity." Cole speculated that, "The existence of chaos in animal behavior can have several important implications. Variation in the temporal component of individual behavior may not be due simply to chance variations in the stochastic world, but to deterministic processes that depend on initial conditions." Inspired by the chaotic and self-organization behaviors of ants, we recently gave a mathematical model called CAS to solve the optimization problems in engineering.

B. Chaotic ant swarm model

Our model searches for optimum or near optimum in the search space symbolized as R^L, the L-dimensional continuous space of real numbers. We consider a population of n ants. These ants are located in a search space S and they try to minimize a function $f:S\rightarrow R$. Each point s in S is a valid solution to the considered problem. In this letter, we only consider the search space to be a continuous space ($S=R^L$). The position of an ant i is the algebraic variable symbol $s_i=(z_{i1},\ldots,z_{iL})$, where $i=1,2,\ldots,n$. Naturally each variable can be of any finite dimension.

In order to obtain the chaotic search initially, the chaotic system, $z(t+1)=z(t)e^{[3-\psi z(t)]}$, is introduced into our equation, and the above system can be obtained from $z(t+1)=z(t)e^{\mu[1-z(t)]}$ which is described by Solé *et al.* in Ref. [20] (for more details please see the Appendix). Obviously during its motion, each individual ant is influenced by their current position, the best position so far by itself and by its neighbors and organization process of the swarm. The adjustment of the chaotic behavior of an individual ant is achieved by the introduction of a successive decrement of organization variable $y_i(t)$ and leads the individual to moving to the new site that acquires the best fitness eventually. To achieve the information exchange of individuals and the movements to the new site taken on the best fitness, we introduce $[p_{id}(t-1)-z_{id}(t-1)]$. The term p_{id} is selected based on the fitness theory which is very widely developed in optimization theory such as genetic algorithm and tabu search, and so on. Thus, we obtain the following detailed dynamical optimization system of chaotic ant swarm:

$$y_i(t)=y_i(t-1)^{(1+r_i)},$$

$$z_{id}(t)=z_{id}(t-1)e^{[1-e^{-ay_i(t)}][3-\psi_d z_{id}(t-1)]}+[p_{id}(t-1)-z_{id}(t-1)]e^{-2ay_i(t)+b}, \tag{1}$$

where a is a sufficiently large positive constant and can be selected as $a=200$, b is a constant and $0\leq b\leq\frac{2}{3}$, ψ_d determines the selection of the search range of the dth element of variable in search space, $r_i\geq 0$ is termed by us as organization factor of ant i, $y_i(0)=0.999$, and $z_{id}(t)$ is the current state of the dth dimension of the individual ant i, where $d=1,2,\ldots,L$. $p_{id}(t-1)$ is the best position found by ith ant and its neighbors within $t-1$ steps, $y_i(t)$ is the current state of the organization variable, t means the current time step, and $t-1$ is the previous step.

$y_i(t)$ and r_i control the convergence of Eq. (1), and the larger the r_i is, the faster the $y_i(t)$ convergences. If the organization factor $r_i=0$, which means the ant swarm is not organized, and in this condition, $e^{-ay_i(t)}$ and $e^{-2ay_i(t)+b}$ approximately equal zero, then Eq. (1) becomes the chaotic model $z_{id}(t)=z_{id}(t-1)e^{[3-\psi_d z_{id}(t-1)]}$; if the organization factor r_i is very large, the time of "chaotic" search is small then the system converges quickly and we cannot achieve the desired optima (or near optima). If the organization factor r_i is very small, the time of chaotic search is large then the system converges slowly and the runtime will be longer. Since small changes are desired as time evolves, the value of r_i is chosen typically as $0<r_i\leq 0.5$. The format of r_i can be designed according to concrete problems and runtime. In order to denote that each ant has different r_i, for example, we can set it as $r_i=0.1+0.2\text{rand}(\ )$, where rand( ) is a uniformly distrib-

uted random number in the interval [0,1]. After the chaotic search, $y_i(t)$ approximately equals zero and the convergence of Eq. (1) will be mainly determined by $z_{id}(t)=z_{id}(t-1)+\exp(b)[p_{id}(t-1)-z_{id}(t-1)]$. In this condition, when $0<b<\ln(2)$, the state $z_{id}(t)$ of Eq. (1) will converge to $p_{id}(t)$. The values r_i should be suitably selected according to the concrete optimization problems.

Via numerous simulations, we find that the above model of chaotic ant swarm [Eq. (1)] searches for optima in constrained positive or negative intervals. That is to say, if $\psi_d>0$, Eq. (1) can be used to realize the search process in the intervals in which all $z_{id}\geq 0$, and if $\psi_d<0$, Eq. (1) can be used to realize the search process in the intervals in which all $z_{id}\leq 0$. When all the optima are located in positive intervals (or negative intervals), Eq. (1) is effective to solve the numerical optimization problems. However, the elements of the optima can be located in all the ranges of real-numbered space. In order to solve the problem of search regions, we introduce $V_i(7.5/\psi_d)$ and give the following version of CAS model which is better than Eq. (1):

$$y_i(t) = y_i(t-1)^{(1+r_i)},$$

$$z_{id}(t) = \left[z_{id}(t-1) + V_i \frac{7.5}{\psi_d} \right] e^{[1-e^{-ay_i(t)}][3-\psi_d z_{id}(t-1)]} + [p_{id}(t-1) - z_{id}(t-1)] e^{-2ay_i(t)+b} - V_i \frac{7.5}{\psi_d}, \quad (2)$$

where $0\leq V_i\leq 1$ determines the search region of ant i and offers the advantage that ants could search diverse regions of the problem space. If $V_i=\frac{1}{2}$, it means the chaotic attractor of ant i moves a half to the negative orientation compared to the chaotic attractor of Eq. (1). The values V_i should be suitably selected according to the concrete optimization problems. We call Eq. (2) the general algorithmic model of chaotic ant swarm. In this model we can select the initial position of an individual ant as $z_{id}(0)=7.5/\psi_d(1-V_i)\text{rand}(\,)$, where $\psi_d>0$. More details about CAS are given in Ref. [14].

ACO algorithms and CAS algorithm are both inspired by ant behaviors. But principles of the two algorithms are different. ACO algorithms, based on probability theory, explain how ants can find the shortest paths between food sources and their nests using pheromone. However, ACO algorithms do not consider the chaotic behaviors of single ants. Chaotic ant swarm algorithm was based on chaotic search strategy and self-organization of ant colony. The search strategies based on chaos properties have been found to obtain nice capabilities of hill climbing and escaping from local optima.

III. DEFINING THE OBJECTIVE FUNCTION

In this section, we will discuss the parameter estimation process of the dynamical systems via chaotic ant swarm algorithm. Let us consider a continuous dynamical system

$$\dot{\vec{x}} = G(\vec{x};\Theta), \quad (3)$$

where $\vec{x}=(x_1,\ldots,x_N)^T\in R^N$ is the state vector of the dynamical system, $\dot{\vec{x}}$ is the derivative of the state vector $\vec{x}$, and $\Theta=(\theta_1,\ldots,\theta_L)$ is the unknown parameter vector of the dynamical system.

The parameter identification process of continuous dynamical system is shown in the following process. Randomly generate the initial positions of all the ants in the searching space, then the initial position of ant i is $\hat{\Theta}_i^0=(\hat{\theta}_{i1}^0,\ldots,\hat{\theta}_{iL}^0)^T$, $i=1,\ldots,K$, where $\hat{\Theta}$ is the estimation of unknown parameter vector Θ. The searching ranges of unknown parameters are selected by ψ_d, where d denotes the search range of the dth element of variable in searching space. Based on the measurable state vector $\vec{x}=(x_1,\ldots,x_N)^T$, we define the following objective function:

$$f(\vec{e}) = \sum_{t=0}^{W} \{[x_1(t) - x_{i1}^n(t)]^2 + \cdots + [x_N(t) - x_{iN}^n(t)]^2\}, \quad (4)$$

where $t=0,1,2,\ldots,W$. Thus, the problem of parameter estimation is transformed to that of using CAS algorithm to search for the suitable value of $\hat{\Theta}_i^n=(\hat{\theta}_{i1}^n,\ldots,\hat{\theta}_{iL}^n)^T$ such that the objective function $f(\vec{e})$ is globally minimized. For each iterative step of the searching algorithm, substitute $\hat{\theta}_i^n$ into Eq. (3). Run the dynamical Eq. (3), then we attain the corresponding state vector $\vec{x}_i^n(t)=[x_{i1}^n(t),\ldots,x_{iN}^n(t)]^T$. At each iterative step, calculate the corresponding objective function by Eq. (4). If the values of the objective function of all the ants meet the desired accuracy or a given maximal number of iterations has been achieved, then we consider that the end condition has been fulfilled. Also the parameter identification process is stopped with the results output.

Now we discuss how to obtain the state vector $\vec{x}(t)$. The simulation was performed in MATLAB by using the well-known fourth-order Runge-Kutta algorithm. Let the dynamical system run freely. After a transient process, select a point randomly as the initial states at which the time is set as zero. Let the dynamical system run from this initial states to Wh. Then we can obtain the standard variable $\vec{x}(t)=[x_1(t),\ldots,x_N(t)]^T$ at time $0h,1h,2h,\ldots,Wh$, where h is the step size.

In order to consider the influence of the noise on the estimation results in practical applications, we define $\vec{\eta}=(\eta_1,\ldots,\eta_N)^T$ as the measurable state vector of dynamical system (3) in the presence of additive observational noise, where $\eta_n=x_n-\xi_n$, ξ_n is an independent and identically distributed sequence, Gaussian random values with zero mean and variance σ_ξ^2. Then the objective function is expressed as

$$f(\vec{e}) = \sum_{t=0}^{W} \{[\eta_1(t) - x_{i1}^n(t)]^2 + \cdots + [\eta_N(t) - x_{iN}^n(t)]^2\}. \quad (5)$$

Through the chaotic ant swarm algorithm, we can derive the estimation $\widehat{\Theta_\xi}$ of parameter Θ in the presence of the additive observational noise. In order to study the effectiveness of the proposed algorithm, we define the following mean-squared error:

$$\varepsilon^2 = \langle (\hat{\theta}_{1\xi} - \theta_1)^2 + \cdots + (\hat{\theta}_{L\xi} - \theta_L)^2 \rangle, \quad (6)$$

where $\langle\cdot\rangle$ stands for mathematical expectation, and $(\langle\hat{\theta}_i\rangle-\theta_i)$, $i=1,\ldots,L$ is the bias in the estimation of θ_i. As is

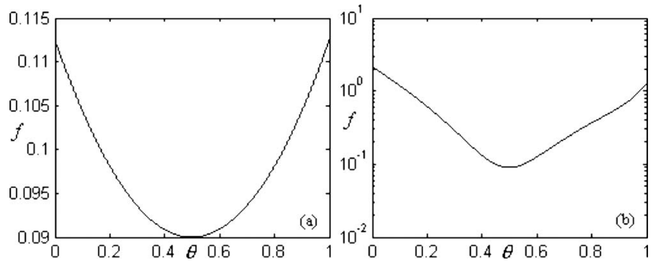

FIG. 1. The graph of the objective function for the time series of stable system (8) at $\theta=0.5, x_0=0.3, h=0.001$: (a) $W=2$; (b) $W=20$.

well known from the theory of probabilities, the mean-squared error can be expressed as

$$\varepsilon^2 = (\langle \hat{\theta}_{1\xi} \rangle - \theta_1)^2 + \cdots + (\langle \hat{\theta}_{L\xi} \rangle - \theta_L)^2 + \sigma^2_{\hat{\theta}_1} + \cdots + \sigma^2_{\hat{\theta}_L}. \tag{7}$$

Thus, we could analyze the estimation results via the mean-squared error. Similarly we could further research on the parameter estimation problem of discrete dynamical systems.

IV. OBJECTIVE FUNCTION BEHAVIOR FOR EXAMPLE MODEL SYSTEMS

In order to study the influence of objective function on the searching performance of the optimization algorithm, we select a linear system and a nonlinear system. For simplicity, both of the dynamical systems are one-dimensional systems with one unknown parameter. The effects of the measurable additive noise on the objective function are also considered. Similarly, we could extend our results to the cases of multi-dimensional systems with multiparameter.

A. One-dimensional linear system

Consider the one-dimensional linear system

$$x_{n+1} = 1 - \theta x_n, \tag{8}$$

where x is the state variable and θ is the unknown parameter.

When the parameter and initial state value of dynamical system (8) are selected as $\theta=0.5$, $x_0=0.3$, system (8) is a stable dynamical system. In Fig. 1, we give the simulation result about the relation between the objective function and the unknown parameter.

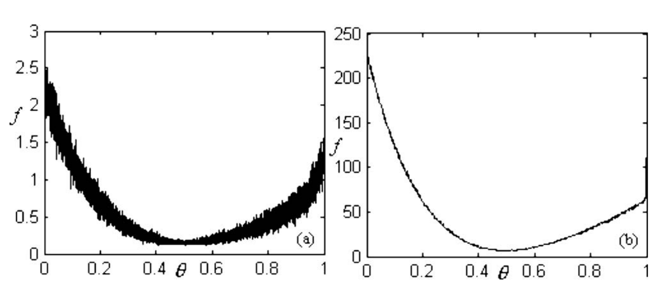

FIG. 2. The graphs of the objective function for the time series of stable system (8) with Gaussian noise of zero mean, 0.01 variance, and 0.1 amplitude and here $\theta=0.5, x_0=0.3, h=0.001$: (a) $W=20$; (b) $W=2000$.

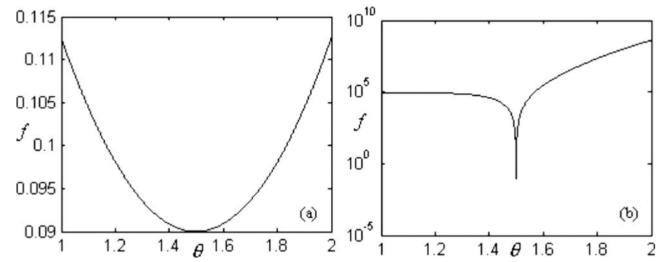

FIG. 3. The graphs of the objective function for the time series of incipiently unstable system (8) at $\theta=1.5, x_0=0.3, h=0.001$: (a) $W=2$; (b) $W=20$.

Seen from Fig. 1, we know that there is only a minimum in the cost function graphs. Through many simulations, we find that the cost function of a one-dimensional linear stable system without noise has one global minimum whatever the length of time series W is increased by. The complexity of cost function does not have a relation with the length of sample series. In this situation it will be easy for the chaotic ant swarm algorithm to find the minimum. But the longer the sample time series is, the longer the time of the estimation is.

In the presence of additive observational noise, Fig. 2 shows the simulation results of the relationships among the objective function, the length time series, and additive observational noise. From Fig. 2, we can see that when W is small (for example, $W=20$), the original time series have been masked by the noise and the global minimum of the objective function does not correspond with the real value of the unknown parameter. Thus, the method based on objective function could not be used to estimate the parameter. But when W is large enough (for example, $W=2000$), the effect of additive noise on the cost function becomes small enough and the method of cost function could be used to estimate the unknown parameter.

When the parameters of dynamical system (8) are selected as $\theta=1.5, x_0=0.3$, system (8) is an incipiently unstable dynamical system. In Fig. 3, we give the simulation results about the relationship between the objective function and the unknown parameter. We can see that Fig. 3 is similar with Fig. 1.

Then we go on our study when there is additive observational Gaussian random noise in the linear incipiently unstable system. In Fig. 4, we give the simulation result about the relationship between the objective function and the unknown parameter, and we could see from Fig. 4 that the case of linear incipiently unstable system is similar with that of the linear stable system. There is only one different point. For the linear incipiently unstable system, if $W \geq 20$, then we could use the method of objective function to estimate the parameter. But for the linear stable system, W should be set larger than 2000.

B. One-dimensional nonlinear system

Consider the following nonlinear system:

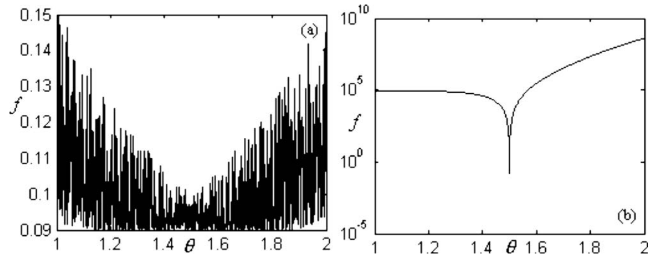

FIG. 4. The graphs of the objective function for the time series of incipiently unstable system (8) with Gaussian noise of zero mean, 0.01 variance, and 0.1 amplitude and here $\theta=1.5, x_0=0.3, h=0.001$: (a) $W=2$; (b) $W=20$.

$$x_{n+1} = 1 - \theta x_n^2, \tag{9}$$

where x is the state variable and θ is the unknown parameter.

When the parameters are selected as $\theta=1.85, x_0=0.3$, system (9) is a chaotic system. In Fig. 5, we give the simulation results about the relationship between the objective function and the unknown parameter.

Seen from Fig. 5, we know that there is only one minimum in the cost function graphs when W is small enough. However, when the length of time series W becomes bigger (for example, $W=20$), the curve of the cost function becomes very complex and there are many local minima in the curve of cost function. When we estimate the unknown parameters of chaotic system, it is evident that the selection of shorter time series is a benefit to the searching process. It is contrary to the traditional view. Generally, it is believed that a longer time series must provide opportunity to get more accurate estimates. When the sample length of the time series becomes longer, the graph of the cost function has many local minima. The cause is that the sensitive dependence of chaotic systems on initial conditions and parameters results in a very complicated cost function graph [21]. This adds the difficulty of the optimization. Thus, many classical optimization methods, such as Newton method and gradient-based method, will not be suitable and the modern heuristic methods used for global optimization will be suitable.

In the presence of additive observational noise, Fig. 6 shows the simulation results about the relationship between the objective function and the unknown parameter. Seen from Fig. 6, when W is small (for example, $W=5$), the original time series of the chaotic system have been masked by the noise, the global minimum of the cost function does not correspond with the real value of the unknown parameter. But when W is a bit larger (for example, $W=20$), the influence of additive noise on the global minimum of the cost function becomes small enough and the global minimum of the cost function corresponds with the real value of the unknown parameter. Though there are many local minima in the graph of the cost function, we still could use the method of cost function to estimate the unknown parameter of the chaotic system.

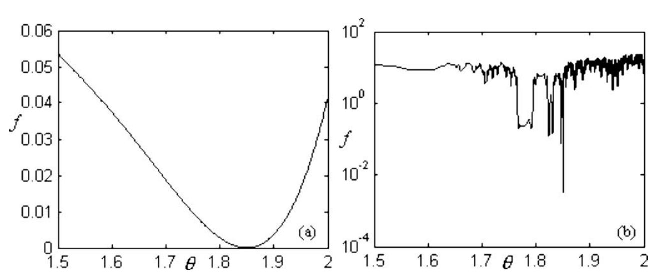

FIG. 5. The graphs of the objective function for the time series of chaotic system (9) at $\theta=1.85, x_0=0.3, h=0.001$: (a) $W=5$; (b) $W=20$.

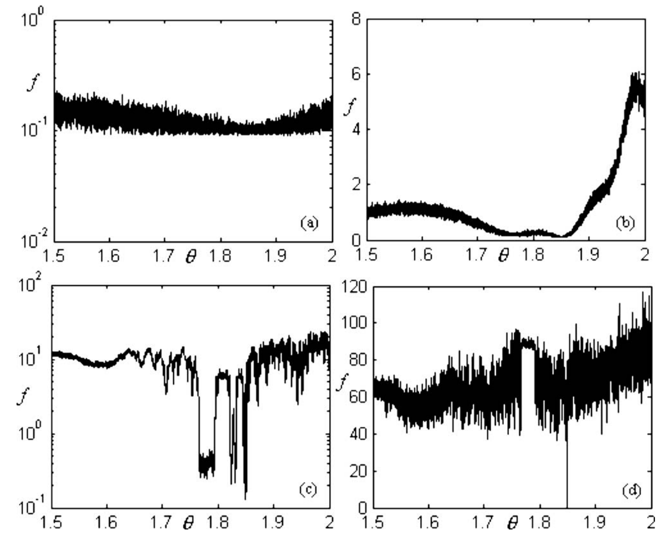

FIG. 6. The graphs of the objective function for the time series of chaotic system (9) with Gaussian noise of zero mean, 0.01 variance, and 0.1 amplitude and here $\theta=1.85, x_0=0.3, h=0.001$: (a) $W=5$, (b) $W=10$, (c) $W=20$, and (d) $W=100$.

When the parameters of dynamical system (9) are selected as $\theta=1.3, x_0=0.9997$, the system is a one-dimensional periodic dynamical system with four periods. The case of periodic system is similar with that of linear stable system, so we omit the corresponding analysis.

Summarizing the results so far, we achieve the following conclusions about the complexity of cost function, the real value of unknown parameter, and the length of time series:

(i) Seen from all the simulation graphs, as the increment of time series length, the cost function value of the real parameter value is departed from those of other parameter values. This agrees with the traditional views that longer time series provide more opportunities to get more accurate estimations.

(ii) When we estimate the unknown parameters of the systems without the influence of noise, the increment of the time series length does not increase the complexity of cost function for the one-dimensional stable, incipiently unstable, and periodic system. But the longer the time series is, the longer the time of the estimation costs. However, for the chaotic system, it is different from the other types of dynamical systems. As the increment of the time series length, although the cost function value of the real parameter value is departed from those of other parameter values, the complexity of cost function is increased. Thus, the selection of suitable series length will benefit the estimation of unknown parameters for the chaotic system.

(iii) In the presence of additive observational noise, when the length of time series is quite small, the cost function

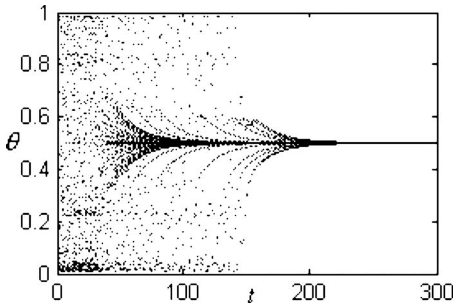

FIG. 7. Estimate of parameter θ in one-dimensional stable linear system, and here, we plot search values of all the ants in order to observe nonlinear dynamical searching process of the ant swarm as a whole.

could not represent the information of the real parameter value. The selection of suitable series length will benefit the estimation of unknown parameters for the dynamical system. For the stable and periodic system, only when the length of time series is very large (for example, $W=2000$), the cost function could represent the information of the real parameter value. For the linear incipiently unstable system, the length of time need not be very large (for example, $W=20$) to estimate the parameter. For the chaotic system, the length of time series need not be very large; the cost function could represent the information of the real parameter value (for example, $W=20$). But the cost function becomes very complicated.

V. PARAMETER ESTIMATION RESULTS FOR EXAMPLE MODEL SYSTEMS

In this section, numerical simulations are given to study the parameter estimation. The simulations of the parameter estimation for dynamical systems with additive noisy time series are also shown in order to verify the conclusions about the relationships between the cost function and time series of dynamical systems in Sec. IV.

A. Parameter estimation results of the noiseless measurable time series

At the beginning, we consider the stable linear system (8) where $\theta=0.5$. Let us set the values of the chaotic ant swarm parameters as $y(0)=0.999$, $a=200$, $b=\frac{2}{3}$, $K=20$, $r_i=0.1+0.3\text{rand}(\,)$ and the search range of the CAS algorithm is $[0,1]$. $W=20$ is the parameter in objective function (4). The result of parameter estimation is shown in Fig. 7. From Fig. 7, we can see that the trajectories of the parameter estimation converge at the real parameter value via the effect of organization variable $y_i(t)$ and organization factor r_i indicating that the model of chaotic ant swarm is very effective to find the real values of unknown parameters for dynamical systems.

For the incipiently unstable linear system and nonlinear periodic system, there is one minimum in the graph of cost function, and the results of estimation are similar with those of the above stable linear system. The corresponding cost function is simple, so the simulation results of estimation are very good. Since these simulation figures are similar with Fig. 7, we do not plan to give concrete figures of these simulations. The results accord with the analysis of Sec. IV.

TABLE I. The estimation results of one-dimensional chaotic system for different values of W.

	$W=30$	$W=40$	$W=60$	$W=100$	$W=120$
1	1.8500	1.8295	1.7064	1.8266	1.8019
2	1.8502	1.8235	1.8294	1.8229	1.6856
3	1.8295	1.8257	1.7069	1.7430	1.8505
4	1.8500	1.8463	1.8502	1.5582	1.6160
5	1.8465	1.8502	1.8463	1.7115	1.6168
6	1.8500	1.7944	1.8583	1.8500	1.8500
7	1.8500	1.8500	1.7070	1.8714	1.6619
8	1.8240	1.8261	1.8550	1.7067	1.5827
9	1.8502	1.8463	1.8257	1.7938	1.8240
10	1.8502	1.7940	1.8481	1.7291	1.8225
11	1.8465	1.7658	1.7963	1.8255	1.7941
12	1.7661	1.8465	1.7071	1.8470	1.8550
13	1.8257	1.8224	1.8505	1.8188	1.7992
14	1.8465	1.8257	1.8235	1.8223	1.5554
15	1.8465	1.7065	1.8556	1.6258	1.7070
16	1.8500	1.8465	1.8242	1.7640	1.7992
17	1.7661	1.7626	1.8255	1.7071	1.8314
18	1.8465	1.8495	1.8257	1.8224	1.8546
19	1.8465	1.8500	1.8485	1.8263	1.8185
20	1.8295	1.8257	1.7989	1.8134	1.6625
ε^2	8.1295×10^{-4}	0.0023	0.0043	0.0119	0.0199
Bias	−0.0140	−0.0306	−0.0405	−0.0757	−0.1006

Then we consider chaotic system (9) where $\theta=1.85, x^0=0.3$. Let us set the values of the chaotic ant swarm parameters as $y(0)=0.999$, $a=200$, $b=\frac{2}{3}$, $K=20$, $r_i=0.05+0.1\text{rand}(\,)$, and the search range of the CAS algorithm is $[1.5,2.5]$. Let W be different values and run the program of chaotic ant swarm algorithm, we use the CAS algorithm to estimate the unknown parameter. From many numerical simulations, we found that the estimation results of parameter θ for the chaotic system were all 1.8500, when W was selected as 2, 5, and 10, respectively. Table I gives the estimation results of the unknown parameter, the corresponding mean-squared error, and bias when W is selected as 30, 40, 60, 100, and 120. Seen from Table I, the mean-squared error ε^2 and $|\text{bias}|$ increase as W increases. The cause is that the critical sensitivity of the chaotic system to initial conditions and parameters results that the cost function becomes very complicated as the increment of W.

B. Simulation results in the presence of additive noise

Under the influence of additive noise, when W is small, the cost function could not represent the information of the real values of unknown parameters for both the systems. Thus, we could not use the method of cost function to estimate unknown parameters. In this section we do not plan to give concrete results of these simulations.

TABLE II. The estimation results of one-dimensional stable linear system for different values of W in the presence of additive noise.

	$W=40$	$W=80$	$W=200$
1	0.5058	0.5072	0.5031
2	0.5232	0.4879	0.5078
3	0.4799	0.5002	0.4886
4	0.4947	0.4988	0.4959
5	0.5016	0.4958	0.5241
6	0.4918	0.5013	0.4979
7	0.5667	0.5167	0.5060
8	0.4953	0.5012	0.5098
9	0.4954	0.4953	0.4924
10	0.4908	0.5027	0.4994
ε^2	5.65×10^{-4}	5.28×10^{-5}	1.04×10^{-4}
Bias	0.0045	0.0007	0.0023
	$W=1000$	$W=2\times10^3$	$W=2\times10^4$
1	0.5032	0.5040	0.4993
2	0.5053	0.5035	0.4996
3	0.5044	0.4994	0.5005
4	0.4996	0.5034	0.4995
5	0.5013	0.4994	0.4988
6	0.5047	0.4965	0.5002
7	0.5040	0.4983	0.5001
8	0.4984	0.4943	0.4995
9	0.5045	0.5005	0.5009
10	0.4959	0.4997	0.500
ε^2	1.79×10^{-5}	8.85×10^{-6}	3.70×10^{-7}
Bias	0.0025	−0.0001	−0.0002

For the stable linear system, only when the length of time series is very large can we estimate the unknown parameters precisely. In the presence of additive Gaussian random noise with zero mean, 0.01 variance, and 0.1 amplitudes, the estimation results using the chaotic ant swarm algorithm are given out in Table II. Seen from Table II, the mean-squared error ε^2 and bias are small when W is bigger enough, and the results are identical with the above analysis about the relationships between the corresponding cost function and measurable series for one-dimensional stable linear system in Sec. IV.

For the incipiently unstable linear system, the length of time series need not be very large, and the unknown parameter could be precisely estimated by the chaotic ant swarm algorithm. The estimation results using the chaotic ant swarm algorithm are given out in Table III. Seen from Table III, when W is quite small, such as $W=2$ and $W=5$, it is difficult to achieve good estimation results. When W is bigger (for example, $W=10$), we could attain estimation results more precisely, especially, when $W\geq20$, the mean-squared error ε^2 and bias are all 0.

Estimation results of the nonlinear periodic system in the presence of noise are similar with those of the stable linear system. When the length of time series is very large, we could estimate the unknown parameters precisely. The simulation results are omitted. For the above three types of dynamical systems, it is evident that if we want to achieve estimation result more precisely when the noise is larger, we need the length of time series to be larger.

TABLE III. The estimation results of one-dimensional incipiently unstable linear system for different values of W in the presence of additive noise.

	$W=2$	$W=5$	$W=10$	$W=20$	$W=50$
1	1.3993	1.5018	1.4994	1.5000	1.5000
2	1.4355	1.5429	1.5028	1.5000	1.5000
3	1.8094	1.5497	1.4990	1.5000	1.5000
4	1.7306	1.5042	1.5011	1.5000	1.5000
5	1.5389	1.4337	1.5025	1.5000	1.5000
6	1.7635	1.5564	1.4965	1.5000	1.5000
7	1.3036	1.5173	1.4981	1.5000	1.5000
8	1.8278	1.4254	1.5033	1.5000	1.5000
9	1.9672	1.4659	1.5027	1.5000	1.5000
10	1.3222	1.5046	1.5008	1.5000	1.5000
ε^2	0.0630	0.0019	5.1340×10^{-6}	0.0000	0.0000
Bias	0.1098	0.0003	0.0006	0.0000	0.0000

Now we begin to consider an example of chaotic systems. For dynamical system (9) where $\theta=1.85, x^0=0.3, h=0.001$. Table IV shows the simulation results of the chaotic system for different values of W when the chaotic system is added with the Gaussian random noise with zero mean, 0.01 variance, and 0.1 amplitudes. From Table IV, we could see that it is difficult to achieve the real value of parameter when the length of time series is very small such as 2 and 5. When W is added to 10 or 20, the estimation results are better, where the mean-squared error ε^2 and bias are small enough. But the estimation results are worse when W is added to be larger than 50. The cause is that the sensitive dependence of chaotic systems on initial conditions and parameters results in a very

TABLE IV. The estimation results of one-dimensional chaotic system for different values of W in the presence of additive noise.

	$W=2$	$W=5$	$W=10$	$W=20$	$W=50$	$W=100$
1	1.8515	1.8822	1.8525	1.8502	1.7943	1.8399
2	1.7758	1.8673	1.8477	1.8500	1.7071	1.8500
3	1.9294	1.9047	1.8538	1.8500	1.7071	1.7293
4	1.9815	1.8463	1.8533	1.8500	1.8297	1.8137
5	1.9649	1.8367	1.8547	1.8502	1.7070	1.5822
6	1.6587	1.8771	1.8460	1.8500	1.8465	1.8260
7	1.9340	1.8694	1.8458	1.8500	1.7946	1.5693
8	1.8678	1.9057	1.8480	1.8500	1.8348	1.8315
9	1.6590	1.8006	1.8472	1.8500	1.7065	1.8249
10	2.0388	1.9070	1.8514	1.8500	1.8235	1.8235
ε^2	0.0158	0.0014	1.0640×10^{-5}	8×10^{-9}	0.0089	0.0169
Bias	0.0161	0.0197	4×10^{-5}	4×10^{-5}	−0.0749	−0.0810

TABLE V. When $W=30$, the estimation results of the chaotic system for different values of additive noise variance σ_ξ^2.

σ_ξ^2	0.001	0.002	0.004	0.006	0.008	0.01
1	1.7940	1.8257	1.8257	1.7940	1.7658	1.7661
2	1.8465	1.7940	1.8257	1.8257	1.8465	1.8257
3	1.8465	1.8465	1.8465	1.7658	1.7940	1.8257
4	1.8500	1.8500	1.7661	1.8465	1.7940	1.8465
5	1.7661	1.8325	1.8308	1.8463	1.7940	1.8500
6	1.8502	1.8295	1.8308	1.8465	1.8295	1.8465
7	1.8500	1.8295	1.8465	1.8222	1.8500	1.7661
8	1.8465	1.7940	1.8257	1.8240	1.8295	1.8500
9	1.8257	1.8465	1.8308	1.8463	1.8465	1.8465
10	1.8295	1.8257	1.8465	1.8465	1.7661	1.8465
11	1.8295	1.8325	1.8502	1.8240	1.8308	1.8500
12	1.8257	1.8325	1.8502	1.8500	1.7940	1.8325
13	1.8465	1.8465	1.8257	1.8257	1.8257	1.8500
14	1.8257	1.8295	1.8295	1.8465	1.8295	1.8500
15	1.8325	1.8465	1.8257	1.8222	1.8295	1.8465
ε^2	0.0013	0.0007	0.0007	0.0010	0.0020	0.0013
Bias	−0.0232	−0.0192	−0.0196	−0.0212	−0.0350	−0.0168

complicated cost function graph. It is identical with the above analysis about the relationships between the corresponding cost function and measurable series for chaotic system in Sec. IV.

We discuss the effect of noise with different amplitudes. When $W=30$, we run the chaotic ant swarm algorithm to estimate the unknown parameter of the chaotic system. The estimation results, the mean-squared error ε^2, and bias are given out in Tables V and VI for different values of noise variance. Seen from Tables V and VI, when the variance of additive noise is changed from 0 to 0.02, the influence of additive noise on the squared error of the simulation results is small.

The error bars of the estimation results in Tables I–VI are shown in Figs. 8 and 9, respectively, where the small "red" dots are data point (D), the big dots denote the data mean (M), the bars with "blue" line show range (R), and the bars with "green" line show standard error (SE) where SE

TABLE VI. When $W=30$, the estimation results of the chaotic system for different values of additive noise variance σ_ξ^2.

σ_ξ^2	0.012	0.014	0.016	0.018	0.02
1	1.8465	1.8465	1.8465	1.8308	1.8257
2	1.8240	1.8295	1.8465	1.8295	1.8500
3	1.8257	1.8257	1.7661	1.8500	1.8222
4	1.8500	1.7661	1.8315	1.7661	1.7944
5	1.8465	1.8325	1.8295	1.8465	1.8315
6	1.8257	1.8295	1.8257	1.7940	1.8295
7	1.8315	1.8325	1.8257	1.8463	1.8240
8	1.8465	1.8315	1.8463	1.8325	1.8500
9	1.8500	1.8465	1.8500	1.8465	1.8465
10	1.8502	1.8395	1.8502	1.7940	1.8257
11	1.8257	1.8295	1.8465	1.8465	1.8295
12	1.8465	1.8465	1.8500	1.8295	1.8502
13	1.8295	1.8465	1.8257	1.8463	1.7661
14	1.7661	1.7661	1.8465	1.8295	1.8500
15	1.8465	1.8465	1.8465	1.8465	1.7661
ε^2	0.0007	0.0016	0.0006	0.0010	0.0014
Bias	−0.0159	−0.0230	−0.0145	−0.0210	−0.0259

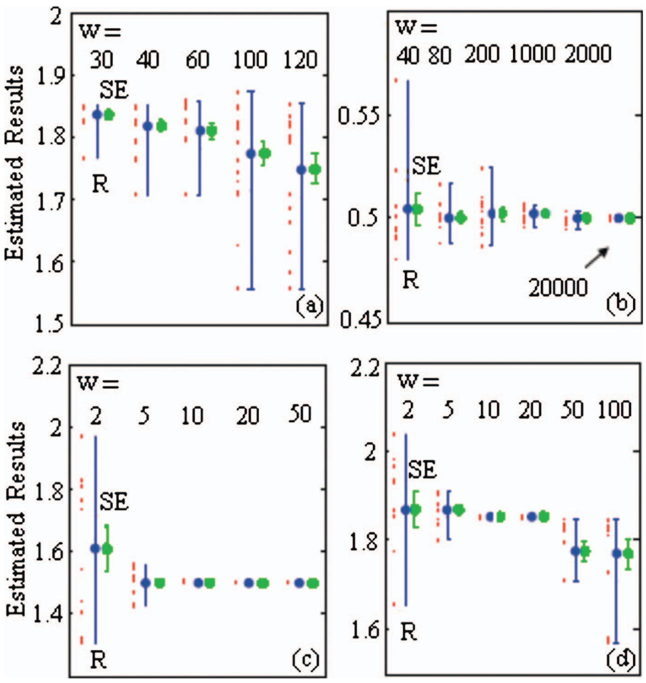

FIG. 8. (Color) The error bars of the estimation results in Tables I–IV, where the small red dots are data point, the bigger blue and green dots represent the data mean (M), the bars with blue line show the range (R), and the bars with green line show the standard error (SE). (a) Error bars of the results in Table I; (b) error bars of the results in Table II; (c) error bars of the results in Table III; (d) error bars of the results in Table IV.

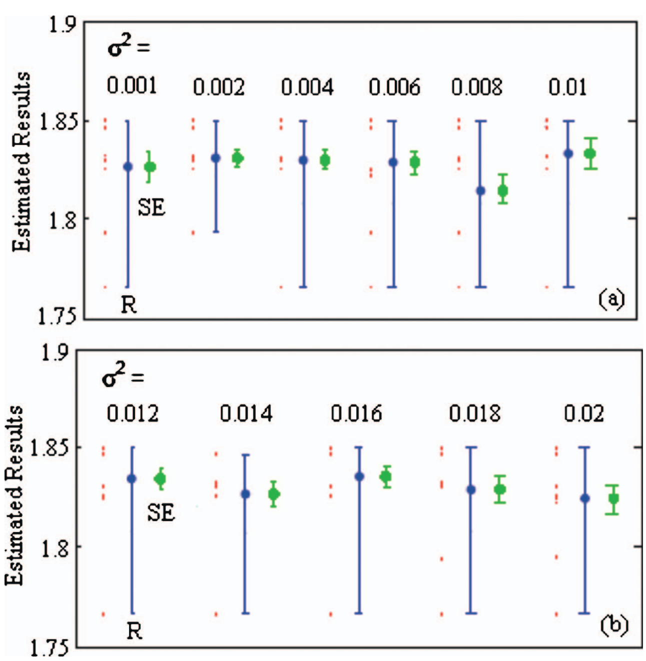

FIG. 9. (Color) The error bars of the estimation results in Tables V and VI, where the small red dots are data point, the bigger blue and green dots represent the data mean (M), the bars with blue line show the range (R), and the bars with green line show the standard error (SE). (a) Error bars of the results in Table V; (b) error bars of the results in Table VI.

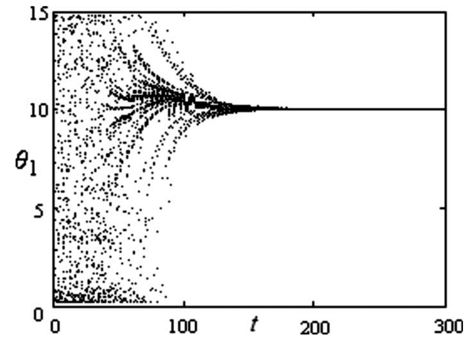

FIG. 10. Estimate of parameter θ_1 of Lorenz chaotic system, and here, we plot search values of all the ants in order to observe nonlinear dynamical searching process of the ant swarm as a whole.

$=\sqrt[2]{\frac{\Sigma(D-M)^2}{n-1}}/\sqrt[2]{n}$. From Figs. 8 and 9, we can see that the shorter the bar lengths of both SE and R are, the better the estimation results are.

VI. SIMULATIONS ON LORENZ SYSTEM

In this section, numerical simulations on Lorenz chaotic system are given to verify the effectiveness and feasibility of the proposed CAS method. The well-known Lorenz system is described by

$$\dot{x}_1 = \theta_1(x_2 - x_1),$$
$$\dot{x}_2 = (\theta_2 - x_3)x_1 - x_2, \tag{10}$$
$$\dot{x}_3 = x_1 x_2 - \theta_3 x_3,$$

where x_1, x_2, and x_3 are the state variables; θ_1, θ_2, and θ_3 are unknown positive constant parameters. The system is in the chaotic state when $\theta_1=10, \theta_2=28, \theta_3=8/3$.

In order to simulate, let the parameters of the Lorenz system be $\theta_1=10$, $\theta_2=28$, $\theta_3=8/3$. Let us set the values of the chaotic ant swarm parameters in Eq. (1) as $y(0)=0.999$, $a=200$, $b=\frac{2}{3}$, $K=20$, $\psi_1=0.5$, $\psi_2=0.15$, $\psi_3=0.75$, $r_i=0.1+0.2\text{rand}(\,)$. $W=30$ is the parameter in objective function (4). The searching ranges for parameters θ_1, θ_2, and θ_3 are [0,5], [0,50], and [0,10], respectively. The estimated process is shown as follows:

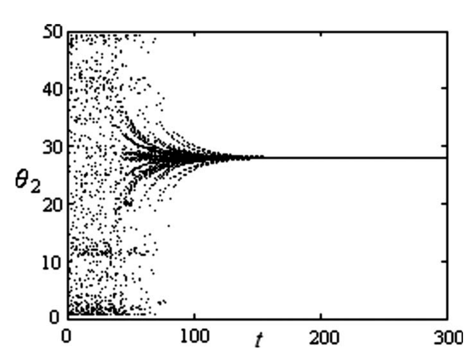

FIG. 11. Estimate of parameter θ_2 of Lorenz chaotic system, and here, we plot search values of all the ants in order to observe nonlinear dynamical searching process of the ant swarm as a whole.

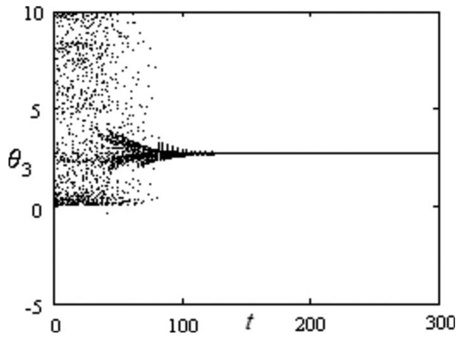

FIG. 12. Estimate of parameter θ_3 of Lorenz chaotic system, and here, we plot search values of all the ants in order to observe nonlinear dynamical searching process of the ant swarm as a whole.

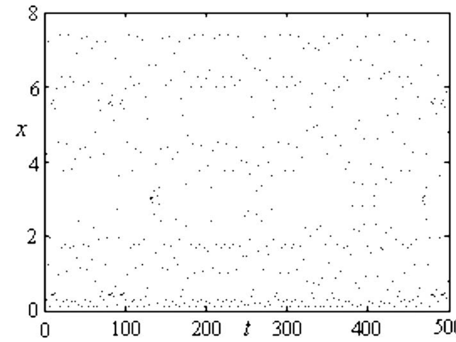

FIG. 13. The chaotic time series of the system $x(t+1) = x(t)e^{[3-x(t)]}$.

(1) Run Lorenz system (10) from initial states to Wh, and then obtain the standard states $\vec{x}(t)=[x_1(t),x_2(t),x_3(t)]^T$ at time $0h,1h,2h,\ldots,Wh$.

(2) Randomly generate the initial positions $\hat{\Theta}_i^n =(\hat{\theta}_{i1}^n,\hat{\theta}_{i2}^n,\hat{\theta}_{i3}^n)^T$ of all the ants in the searching space $[0,7.5/\psi_d], d=1,2,3$.

(3) Substitute $\hat{\Theta}_i^n$ into Lorenz system (10), and run system (10) to obtain the states $\vec{x}_i^n(t)=[x_{i1}^n(t),x_{i2}^n(t),x_{i3}^n(t)]^T$ at time $0h,1h,2h,\ldots,Wh$.

(4) Calculate the objective function $f(\vec{e})=\sum_{t=0}^{W}\{[x_1(t)-x_{i1}^n(t)]^2+[x_2(t)-x_{i2}^n(t)]^2+[x_3(t)-x_{i3}^n(t)]^2\}$.

(5) Search based on system (1) of CAS.

(6) Let $n=n+1$ and go to step 3 until the evaluation function is smaller than the specified precision or the iteration satisfies a specified maximal iterative step.

The results of parameters estimation are shown in Figs. 10–12. From Figs. 10–12, we can see that the trajectories of the estimation of the parameters converge at the real values of parameters and the chaotic ant swarm algorithm can be used as an effective parameter estimation method.

Table VII gives the simulation results of the Lorenz chaotic system for different values of noise variance σ_ξ^2 when $W=20$. Each result is the average over ten times in Table VII. Seen from Table VII, the results of the mean-squared error ε^2 are small in the whole when the values of noise variance change from 0 to 1. It indicated that the estimation results are ideal. If we want to achieve the estimation results of higher accuracy, we could improve in the following three ways:

(1) Run many times the algorithm. For example, each experiment runs 30 times. The maximum and minimum of these 30 trials are selected as the new range of the searching. Then run the program of the algorithm in the new range to search for the real values of unknown parameters.

(2) For the chaotic ant swarm algorithm, the chaotic searching time is controlled by the organization variable $y_i(n)$ and organization factor r_i. So we could decrease organization factor r_i appropriately and increase the iterative times of searching to improve the accuracy of searching.

(3) Increase the number of ants in the chaotic ant swarm algorithm. Because the CAS algorithm is based on the swarm searching metaheuristic, the increment of scale of the swarm will lead to higher accuracy.

VII. CONCLUSION

In this paper, we succeeded to use the chaotic ant swarm method for estimating the unknown parameters of the dynamical systems. We systematically analyzed the relationships between the complexity of cost function and the length of time series when the dynamical system is in one of the four states—stability, incipiently unstable, periodicity, and chaos. The effects of the measurable additive noise on the cost function were also studied. The simulations verified the effectiveness and feasibility of the proposed algorithm. Through numerical simulations, we achieved the following conclusions: for the stable, periodic, and incipiently unstable systems, longer length of time series will benefit the parameter estimation; for the chaotic system without the influence of noise, shorter length of time series will benefit the parameter estimation because that longer series length will make the cost function complicated; while for the chaotic system in presence of noise, certain length of time series will make the CAS algorithm converge well in the process of estimation. The present method often produces biased estimates when nonlinear dynamics is involved. Since the different values of time series length W correspond to different esti-

TABLE VII. The average results of mean-squared error ε^2 for different values of noise variance σ_ξ^2.

σ_ξ^2	0.00	0.05	0.10	0.15	0.20	0.25	0.30
ε^2	0.0000	0.0143	0.0128	0.0708	0.0134	0.0482	0.4829
σ_ξ^2	0.35	0.40	0.45	0.50	0.55	0.60	0.65
ε^2	2.0000	0.2482	0.1085	1.2483	0.0321	0.2947	0.0769
σ_ξ^2	0.70	0.75	0.80	0.85	0.90	0.95	1.00
ε^2	0.0185	0.0476	0.1618	0.1787	0.0583	0.0200	0.0318

mated bias, the selection of suitable series length could reduce the bias. There may be some outliers in the data set of estimation results. The Grubbs test could be used to test for outliers in a univariate data set. Deleting the outliers in the data set for the estimation results may be used to reduce the bias. Moreover, there are some further significant directions to be investigated such as parameter identification of spatiotemporal chaotic systems and topological structures identification of complex networks based on optimization methods.

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APPENDIX

In order to describe the chaotic search of ants, we introduce the chaotic model $z(t+1)=z(t)e^{\mu[1-z(t)]}$. Let $z(t)=\frac{1}{\mu}x(t)$; we have $x(t+1)=x(t)e^{[\mu-x(t)]}$. When $\mu=3$, the system is in chaotic state, which is shown in Fig. 13, and here the search center of $x(t)$ is approximately 7.5/2. Let $x(t)=\psi z(t)$, then we obtain the model $z(t+1)=z(t)e^{[3-\psi z(t)]}$.

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